ISSN-2455-0736 (Print)
ISSN-2456-4052 (Online)

A Comparative Study on Markowitz Mean-Variance Model and Sharpe's Single Index Model in the Context of Portfolio Investment

Josmy Varghese [1] and Anoop Joseph [2]

Department of Commerce, Pavanatma College, Murickassery [1]
Department of Commerce, Pavanatma College, Murickassery [2]
E-Mail: josmyve@gmail.com [1], anoopj.pcm@gmail.com [2]

Article History

Received:
15 March 2018
Received in revised form: 30 April 2018
Accepted:
15 May 2018

KEY WORDS:
expected return, risk, beta, investment, portfolio, model

ABSTRACT

Investment is allocating resources into something with the expectation of gain, usually over a longer term. It is like postponed consumption as defined by Fuller and Farrell. A portfolio refers to a collection of assets or investment instruments depending on the investor's income, budget and convenient time frame. Portfolio management refers to the selection of assets and their continuous shifting in the portfolio for optimizing the return and maximizing the wealth of an investor by analyzing the strengths, weaknesses, opportunities, and challenges for performing a wide range of activities related to a portfolio. Portfolio management is a process which is somewhat continuous in nature as that the steps involved are repeating after a full cycle. One of the steps in portfolio management is the portfolio analysis with regard to the risks and returns attached to a particular portfolio. In the course of portfolio analysis, investors are supposed to do select the optimum portfolio in all respects, i.e., selecting the low risk and high rewarding portfolio. It helps in making selection of various tradeoffs. The present study highlights a comparison between two famous models used for portfolio analysis viz, Markowitz model and Sharpe's model.

1.0 Introduction to Portfolio Analysis

The term 'investment' is used differently in economics and in finance. In finance, investment is allocating resources into something with the expectation of gain, usually over a longer term. In the words of economists "investment is the net additions to the economy's capital stock which consists of goods and services that are used in the production of other goods and services".

A portfolio refers to a collection of assets or investment tools such as stocks, shares, mutual funds, bonds, and cash and so on depending on the investor's income, budget and convenient time frame. Portfolio management refers to the selection of securities and their continuous shifting in the portfolio for optimizing the return and maximizing the wealth of an investor. It also refers to the science of analyzing the strengths, weaknesses, opportunities, and challenges for performing a wide range of activities related to one's portfolio for maximizing the return at a given risk. It helps in making a selection of debt vs. equity, growth vs. safety, and various other tradeoffs. It aims at managing various investments of individuals to earn the maximum profits within the stipulated time frame. In this regard, it can be said that portfolio management is the art of selecting the right investment policy for the individuals in terms of minimum risk and maximum return.

1.1 Importance of Portfolio Investment

1.2 A portfolio investment is a liberal or passive investment of securities in a portfolio, and it is made with the expectation of earning a return. This expected return is directly correlated with the investment's expected risk. Investment through portfolio is separate from direct investment, which involves taking a substantial stake in a target company in its true sense. Portfolio investments can cover a wide range of asset classes. The composition of investments in a portfolio depends on a number of factors such as the investor's risk tolerance, investment horizon and amount invested, etc. Investing in portfolio rather than in individual asset gains attraction because of its 'risk reduction and performance optimization' capability. In order to design an outstanding portfolio, the investor needs to take vital decisions which can influence the very performance of the portfolio. The present study light up the influential factors which can affect the decisions taken in the context of a portfolio and tries to emphasis the dire need of having an investment portfolio.

1.2 Some Historical Backdrop

Harry Markowitz ("Markowitz") is highly regarded as a pioneer for his theoretical contributions to financial economics and corporate finance. In 1990, Markowitz shared a Nobel Prize for his contributions to these fields, espoused in his "Portfolio Selection" (1952) essay first published in The Journal of Finance, and more extensively in his book, "Portfolio Selection: Efficient Diversification (1959). His groundbreaking work formed the foundation of what is now popularly known as 'Modern Portfolio Theory' (MPT). The foundation for this theory was substantially later expanded upon by Markowitz' fellow Nobel Prize co-winner, William Sharpe, who is widely known for his 1964 Capital Asset Pricing Model work on the theory of financial asset price formation.

1.3 Objectives of the Study

1. To compare the portfolio analysis models suggested by Markowitz and Sharpe.
2. To list out the benefits for the investors from using these models.

1.4 Methodology

The study aims at providing some thoughts on mean-variance model suggested by Harry Markowitz and single index model suggested by William Sharpe. It compares these two popular models used for portfolio analysis. The study is descriptive in nature. All the relevant information are collected using books, articles, websites, and few interviews with experts in the field of portfolio investment.

1.5 Markowitz Mean-Variance Model of Portfolio Analysis

Most people agree that holding two stocks is less risky than holding one stock. As per the model introduced by Harry Markowitz, the expected return of a portfolio of securities is the weighted average expected return of its component securities. The proportion of the component securities in the current value of the portfolio is used as weight. In symbols, the general rule for calculating the expected return of a portfolio consisting of 'n' securities may be expressed as under:

$$\overline{R}_p = \sum_{i=1}^{n} X_i \overline{r}_i$$

Where, $\overline{R}_p$ = Expected return of portfolio, X_i = Proportion of funds invested in each security (or, Security i), $\overline{r}_i$ = Expected return of each security (or, Security i), n = Number of securities in the portfolio.

As per the Markowitz approach portfolio risk (σ_p) is the sum total of the risk of individual securities included in the portfolio and the interactive risk in combining them. The risk involved in individual securities can be measured by standard deviation (σ) or variance (σ^2). σ or σ^2 tells the variability of actual return from the expected return. In a portfolio context it is not possible to measure the riskiness of a portfolio simply by adding together the weighted average risk of the component securities. To measure the portfolio risk, in addition to the risk of individual securities, we should also consider the interactive risk between the securities. Interactive risk can be measured by covariance (σ_{ij}) or (Cov_{xy}).

Therefore, *Portfolio risk = Weighted average of risk of individual securities + Interactive risk*.

Interactive risk or covariance is a statistical measure that indicates the interactive risk of a security relative to other securities in a portfolio of securities.

The variance of a portfolio with only two securities may be calculated as:

$$\sigma_p^2 = x_1^2 \sigma_1^2 + x_2^2 \sigma_2^2 + 2x_1x_2 (r_{12} \sigma_1 \sigma_2)$$

Where, σ_p^2 = Portfolio variance

x_1 = Proportion of funds invested in the first security

x_2 = Proportion of funds invested in the second security

σ_1^2 = Variance of first security

σ_2^2 = Variance of second security

σ_1 = Standard deviation of first security

σ_2 = Standard deviation of second security

r_{12} = Correlation coefficient between the returns of first and second security

$r_{12} = (Cov_{12}) \div (\sigma_1 \sigma_2)$

$Cov_{12} = \{\sum_{i=1}^{n}[R_1 - \overline{R}_1]*[R_2 - \overline{R}_2]\} \div N$

Where, Cov_{12} = Covariance between x and y

R_1 = Return of security 1

R_2 = Return of security 2

$\overline{R}_1$ = Expected or mean return of security 1

$\overline{R}_2$ = Expected or mean return of security 2

N = Number of observations

Portfolio standard deviation (σ_p) can be obtained by taking the square root of the portfolio variance.

1.6 Sharpe's Single Index Model of Portfolio Analysis

As per this model, founded by William Sharpe, expected return of the portfolio is the weighted average of the market related and non-market related component of the expected return of the individual security.

Thus, $\overline{R}_p = \sum_{i=1}^{n} X_i \beta_i \overline{r}_m + \sum_{i=1}^{n} X_i \alpha_i$

Where, $\overline{R}_p$ = Expected portfolio return

X_i = Proportion of investment in 'i'th security

β_i = Beta value or systematic risk of the 'i'th security

(Beta of stock 'i' = [(Correlation coefficient between the returns of stock 'i' and the returns of the market index) × (Standard deviation of returns of stock 'i') × (Standard deviation of returns of the market index)] ÷ Variance of the market returns.

Symbolically, $\beta_i = (r_{im}\, \sigma_i\, \sigma_m) \div \sigma_m^2$

$\overline{r}_m$ = Expected return of the market index

α_i = Unsystematic or non-market related return on 'i'th security

According to the Sharpe's model, the risk of a portfolio can be expressed in terms of the risk of individual securities in the portfolio i.e., the total risk of the portfolio accounts for market risk and unique risk.

$\sigma_p^2 = \beta_p^2\ \sigma_{rm}^2 + \sigma_{ep}^2$ ***or*** $\sigma_p^2 = [\sum_{i=1}^{n}(X_i\, \beta_i)^2\ \sigma_{rm}^2]+[\ [\sum_{i=1}^{n} (X_i^2\sigma_{ei}^2)]$

Where, σ_p^2 = Variance of the portfolio

β_p^2 = Weighted average betas squared of individual securities in the portfolio

σ_{rm}^2 = Variance of the return on market index

σ_{ep}^2 = Weighted average of the residual variance of securities in the portfolio (weighted variance of securities' return not related to market index)

(Weights being proportion of funds invested in securities in the portfolio (Xi))

1.7 Comparison between Markowitz and Sharpe Model

The Markowitz model shows that if a portfolio consists of N securities, N pieces of information relating to expected return, N pieces of information relating to variances, and $(N^2 -N)/2$ pieces of covariance terms are needed for a given proportion of N securities in the portfolio. Thus, total of [N (N+ 3) /2] separate pieces of information are needed before efficient portfolios are identified for a given proportion of N securities. E.g. suppose there are 10 securities in a portfolio. Then separate pieces of information needed to build an efficient portfolio are 65 pieces i.e., [10(10+3)/2].

In other words, 10 pieces of expected returns plus 10 pieces of variances plus $(10^2-10)/2$ pieces of covariance equal to 65 pieces.

In the Single Index model, equations of portfolio return and risk shows that the expected return and risk of a portfolio can be estimated if we have estimate of beta for each stock, alpha for each stock, an estimate of unique risk for each stock and finally an estimate of both expected return of market index ($\overline{r}_m$) and variance on the return of market index (σ_{rm}^2) i.e., expected return and variance for any approved market index. Thus total of (3N + 2) pieces of information are required for risk return analysis. E.g. suppose there are 10 securities in a portfolio. Then separate pieces of information needed to build an efficient portfolio are 32 pieces i.e., (3*10+2).

In other words, 10 pieces of beta plus 10 pieces of alpha plus 10 pieces of unique risk plus one piece of expected return of market index and one piece of variance on the return of market index equals 32 pieces.

This makes Sharpe's Single index Model superior to Markowitz mean-variance model. Furthermore there is no need for computing the covariance of each possible combination of securities. Only estimates of the way each security moves with the market is sufficient.

Again, Markowitz model shows that if perfectly negatively correlated securities' inappropriate proportion is included in the portfolio, portfolio risk can be reduced to the zero level. But practically, it is impossible to identify securities with perfect negative correlation.

Sharpe's view of reduction of risk through diversification is that diversification results in the reduction of unique risk of the portfolio. The market risk of the portfolio cannot be reduced through diversification.

1.8 Conclusion

This paper attempts to compare the portfolio analysis models suggested by Harry Markowitz and William Sharpe and is intended for brief study of these models. The study was conducted according to the plan proposed. The important thing to remember is that the models are just tools although perhaps the biggest hammer in one's financial toolkit. It has been nearly sixty years since Markowitz first expounded on mean-variance model and it is unlikely that its popularity will fade anytime in the near future. His theoretical conclusions have become the spur for the development of other theoretical analysis in the field of portfolio theory. Even so, Markowitz' model is dependent upon continued probabilistic growth and expansion. Sharpe's Single Index Model is very useful to construct an optimal portfolio by analyzing how and why securities are included in an optimal portfolio, with their respective weights calculated on the basis of some important variables under consideration. One of the most important limitations with Sharpe's Single Index model is that it does not consider uncertainty in the market as time progresses; instead the model optimizes for a single point in time. Moreover, this model assumes that security prices move together only because of common co-movement with the market. Many researchers have identified that there are influences beyond the general business and market conditions. However, empirical evidence shows that the more complicated models have not been in a position to outperform the single index model in terms of their ability to predict ex -ante co-variances between security returns. The study was descriptive in nature. The study has identified a number of similarities and differences of the two models which can influence the decisions taken with regard to the investment portfolio.

1.9 Limitations

Despite its momentous theoretical importance, there are numerous critics of these two models who argue that its underlying assumptions and modeling of financial markets are not in line with the real world in many ways. Beginning with the key assumptions itemized at the beginning of this analysis, it can be argued that none of these assumptions are entirely true, and that each of them, to varying degrees, compromises the very use of these models.

1.10 Implications

A study about the highlights of mean-variance model suggested by Harry Markowitz and single index model suggested by William Sharpe is practically somehow a waste of time. In practice, no investor can be patient of choosing an optimal portfolio using the tedious or cumbersome process involved in these models. Moreover they are investing in securities based on availability and short term gains in mind.

1.11 Scope for Further Research

There must be a single model which combines the advantages of different models of portfolio and (or) security analysis. Besides this, common investors must have minimum efforts in the portfolio decision making to construct their own efficient portfolio.

BIBLIOGRAPHY

1 Directorate of Studies. (2014). Advanced Financial Management. *Study Note*, 2. Kolkata: The Institute of Cost Accountants of India (ICAI).
2 Anoop Joseph. (2016). Influential Factors of Company Analysis in the Context of Investment. PESQUISA, 2(1).
3 Anoop Joseph & Josmy Varghese. (2017). A Study on Factors Affecting Investment Decision Making in the Context of Portfolio Management. PESQUISA, 3(1).
4 Chandra Prasanna. (2009). Investment Analysis and Portfolio Management (2 ed.). New Delhi: Tata McGrew-Hill.
5 Fuller, Russell J., Farrell and James L. (1987). Modern Investment and Security Analysis. New York: McGraw Hill.
6 Myles E. Mangram. (2013). A Simplified Perspective of the Markowitz Portfolio Theory. Global Journal of Business, 7.
7 Niranjan Mandal. (2013). SHARPE'S SINGLE INDEX MODEL AND ITS APPLICATION TO CONSTRUCT OPTIMAL PORTFOLIO: AN EMPIRICAL STUDY. Great Lakes Herald, 7(1).
8 Portfolio Management. (2007). *Study Note*. Hyderabad: ICFAI University Press.
9 Pandian, Punithavathy. (2009). Security Analysis and Portfolio Management. Noida: Vikas Publishing House Pvt Ltd.
10 Ranganatham M.(2005). Investment analysis and portfolio management. Delhi: Pearson Education (P) Ltd.
11 Rustagi R P. (2005). Investment Management – Theory & Practice. (1 ed.). New Delhi: Sultan Chand & Sons.